

Thermodynamical description of interacting entropy-corrected new agegraphic dark energy

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Abstract

To explain the accelerating universe driven by dark energy, a so-called “entropy-corrected new agegraphic dark energy” (ECNADE), was recently proposed with the help of quantum corrections to the entropy-area relation in the framework of loop quantum cosmology. Using this definition, we study its thermodynamical features including entropy and energy conservation. We discuss the thermodynamical interpretation of the interaction between ECNADE and dark matter in a non-flat universe bounded by the apparent horizon. We obtain a relation between the interaction term of the dark components and thermal fluctuation.

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1 Introduction

Among the overflowing and complementary candidates to explain the cosmic acceleration, the agegraphic and new agegraphic dark energy (DE) models condensate in a class of quantum gravity may have interesting cosmological consequences. These models take into account the Heisenberg uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. The agegraphic DE (ADE) models assume that the observed DE comes from the spacetime and matter field fluctuations in the universe [1, 2, 3]. Since in ADE model the age of the universe is chosen as the length measure, instead of the horizon distance, the causality problem in the holographic DE (HDE) is avoided. The ADE models have been examined and constrained by various astronomical observations [4, 6, 5, 7]. Although going along a fundamental theory such as quantum gravity may provide a hopeful way towards understanding the nature of DE, it is hard to believe that the physical foundation of ADE is convincing enough. Indeed, it is fair to say that almost all dynamical DE models are settled at the phenomenological level, neither HDE model nor ADE model is exception. Though, under such circumstances, the models of HDE and ADE, to some extent, still have some advantage comparing to other dynamical DE models because at least they originate from some fundamental principles in quantum gravity.

Besides, in the loop quantum gravity, the entropy-area relation can modify due to the thermal equilibrium fluctuations and quantum fluctuations [8, 9, 10]. The corrected entropy takes the form [11]

$$S = \frac{A}{4G} + \tilde{\alpha} \ln \frac{A}{4G} + \tilde{\beta}, \quad (1)$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are two dimensionless constants of order unity. Motivated by the corrected entropy-area relation (1) the energy density of the ECNADE was proposed as [12]

$$\rho_{\Lambda} = \frac{3n^2 M_P^2}{\eta^2} + \frac{\alpha}{\eta^4} \ln(M_P^2 \eta^2) + \frac{\beta}{\eta^4}, \quad (2)$$

where α and β are dimensionless constants of order unity. Also the conformal time η is given by

$$\eta = \int \frac{dt}{a} = \int_0^a \frac{da}{Ha^2}. \quad (3)$$

The motivation idea for taking the energy density of ECNADE in the form (2) comes from the fact that both ADE and HDE models have the same origin. Indeed, it was argued that the ADE models are the HDE model with different IR length scales [13]. In the special case $\alpha = \beta = 0$, Eq. (2) yields the energy density of NADE in Einstein gravity [2].

Our main purpose in this paper is to study thermodynamical picture of the interaction between dark matter (DM) and ECNADE model for a universe with spacial curvature. Thermodynamical description of the interaction (coupling) between HDE and DM has been studied in [14, 5, 15]. The paper is outlined as follows. In the next section we consider the thermodynamical picture of the non-interacting ECNADE in a non-flat universe. In section 3, we study the thermodynamical description in the case where there is an interaction term between the dark components. We also present an expression for the interaction term in terms of a thermal fluctuation. We conclude our paper in section 4.

2 Thermodynamical description of the non-interacting ECNADE

We start with the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe which is described by the line element

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (4)$$

where $a(t)$ is the scale factor and k is the curvature parameter with $k = -1, 0, 1$ corresponding to open, flat, and closed universes, respectively. The first Friedmann equation governing the evolution of the universe is

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2} (\rho_\Lambda + \rho_m), \quad (5)$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ_m and ρ_Λ are the energy density of DM and DE, respectively. The fractional energy densities are defined as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3M_P^2 H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\rho_\Lambda}{3M_P^2 H^2}, \quad \Omega_k = \frac{k}{a^2 H^2}. \quad (6)$$

Thus, we can rewrite the first Friedmann equation as

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k. \quad (7)$$

From Eq. (2) the fractional energy density of the ECNADE can be written as

$$\Omega_\Lambda = \frac{n^2}{H^2 \eta^2} \gamma_n, \quad (8)$$

where

$$\gamma_n = 1 + \frac{1}{3n^2 M_P^2 \eta^2} [\alpha \ln(M_P^2 \eta^2) + \beta]. \quad (9)$$

The continuity equations for DE and pressureless DM are

$$\dot{\rho}_\Lambda^0 + 3H_0(1 + \omega_\Lambda^0)\rho_\Lambda^0 = 0, \quad (10)$$

$$\dot{\rho}_m^0 + 3H_0\rho_m^0 = 0, \quad (11)$$

where ω_Λ^0 is the equation of state (EoS) parameter of ECNADE when it evolves independently of DM. The superscript/subscript “0” denotes that there is no interaction between the dark components and in this picture our universe is in a thermodynamical stable equilibrium.

The equation of motion of Ω_Λ^0 is [12]

$$\Omega_\Lambda'^0 = \Omega_\Lambda^0 \left[3(1 - \Omega_\Lambda^0) + \Omega_k^0 + \frac{2}{na_0} \left(\frac{\Omega_\Lambda^0}{\gamma_n^0} \right)^{1/2} \left(2\gamma_n^0 - 1 - \frac{\alpha H_0^2}{3M_P^2 n^4} \frac{\Omega_\Lambda^0}{\gamma_n^0} \right) \left(\frac{\Omega_\Lambda^0 - 1}{\gamma_n^0} \right) \right], \quad (12)$$

where the prime stands for the derivative with respect to $x^0 = \ln a_0$. Taking the time derivative of Eq. (2) and using Eq. (8) we get

$$\dot{\rho}_\Lambda^0 = -\frac{2H_0}{na_0 \gamma_n^0} \left(\frac{\Omega_\Lambda^0}{\gamma_n^0} \right)^{1/2} \left(2\gamma_n^0 - 1 - \frac{\alpha H_0^2}{3M_P^2 n^4} \frac{\Omega_\Lambda^0}{\gamma_n^0} \right) \rho_\Lambda^0. \quad (13)$$

Substituting into Eq. (10) we easily find the EoS parameter of the non-interacting ECNADE

$$1 + \omega_\Lambda^0 = \frac{2}{3na_0\gamma_n^0} \left(\frac{\Omega_\Lambda^0}{\gamma_n^0} \right)^{1/2} \left(2\gamma_n^0 - 1 - \frac{\alpha H_0^2}{3M_P^2 n^4} \frac{\Omega_\Lambda^0}{\gamma_n^0} \right). \quad (14)$$

We also assume the local equilibrium hypothesis to be hold. This requires that the temperature T of the energy content inside the apparent horizon should be in equilibrium with the temperature T_h associated with the apparent horizon, so we have $T = T_h$. If the temperature of the fluid differs much from that of the horizon, there will be spontaneous heat flow between the horizon and the fluid and the local equilibrium hypothesis will no longer hold. This is also at variance with the FRW geometry. Thus, when we consider the thermal equilibrium state of the universe, the temperature of the universe is associated with the horizon temperature. The equilibrium entropy of the ECNADE is connected with its energy and pressure through the first law of thermodynamics

$$T_0 dS_\Lambda^0 = dE_\Lambda^0 + P_\Lambda^0 dV_0, \quad (15)$$

where the volume enveloped by the apparent horizon is given by

$$V_0 = \frac{4\pi}{3} (r_A^0)^3, \quad (16)$$

and r_A^0 is the apparent horizon radius of the FRW universe

$$r_A^0 = \frac{1}{\sqrt{H_0^2 + k/a_0^2}}. \quad (17)$$

The equilibrium energy of the ECNADE inside the apparent horizon is

$$E_\Lambda^0 = \rho_\Lambda^0 V_0 = \left[3n^2 M_P^2 \eta_0^{-2} + \alpha \eta_0^{-4} \ln(M_P^2 \eta_0^2) + \beta \eta_0^{-4} \right] \left[\frac{4\pi}{3} (r_A^0)^3 \right]. \quad (18)$$

Taking the differential form of Eq. (18) and using Eq. (8), we find

$$dE_\Lambda^0 = -12\pi (r_A^0)^3 M_P^2 H_0^2 \Omega_\Lambda^0 \left[(1 + \omega_\Lambda^0) dx^0 - \frac{dr_A^0}{r_A^0} \right]. \quad (19)$$

The associated temperature on the apparent horizon can be written as

$$T_0 = \frac{1}{2\pi r_A^0}. \quad (20)$$

Finally, combining Eqs. (19) and (20) with Eq. (15) we obtain

$$dS_\Lambda^0 = 24\pi^2 (r_A^0)^3 M_P^2 H_0^2 \Omega_\Lambda^0 (1 + \omega_\Lambda^0) \left[dr_A^0 - r_A^0 dx^0 \right]. \quad (21)$$

3 Thermodynamical description of the interacting ECNADE

Next we generalize our study to the case where the pressureless DM and the ECNADE interact with each other. In this case ρ_m and ρ_Λ do not conserve separately; they must rather enter the energy balances [16]

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \quad (22)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (23)$$

where Q is the interaction term. Inserting Eq. (13) without superscript/subscript “0” into (22), we obtain the EoS parameter of the interacting ECNADE

$$1 + \omega_\Lambda = \frac{2}{3na\gamma_n} \left(\frac{\Omega_\Lambda}{\gamma_n} \right)^{1/2} \left(2\gamma_n - 1 - \frac{\alpha H^2}{3M_P^2 n^4} \frac{\Omega_\Lambda}{\gamma_n} \right) - \frac{Q}{9M_P^2 H^3 \Omega_\Lambda}. \quad (24)$$

The evolution behavior of the ECNADE is now given by [12]

$$\begin{aligned} \Omega'_\Lambda = \Omega_\Lambda & \left[3(1 - \Omega_\Lambda) - 3b^2(1 + \Omega_k) + \Omega_k \right. \\ & \left. + \frac{2}{na} \left(\frac{\Omega_\Lambda}{\gamma_n} \right)^{1/2} \left(2\gamma_n - 1 - \frac{\alpha H^2}{3M_P^2 n^4} \frac{\Omega_\Lambda}{\gamma_n} \right) \left(\frac{\Omega_\Lambda - 1}{\gamma_n} \right) \right]. \end{aligned} \quad (25)$$

As soon as an interaction between dark components is taken into account, they cannot remain in their respective equilibrium states. The effect of interaction between the dark components is thermodynamically interpreted as a small fluctuation around the thermal equilibrium. Therefore, the entropy of the ECNADE is connected with its energy and pressure through the first law of thermodynamics

$$TdS_\Lambda = dE_\Lambda + P_\Lambda dV, \quad (26)$$

where

$$T = \frac{1}{2\pi r_A}, \quad (27)$$

$$V = \frac{4\pi}{3} r_A^3, \quad (28)$$

and

$$r_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \quad (29)$$

Now we have an extra logarithmic correction term in the entropy expression

$$S_\Lambda = S_\Lambda^{(0)} + S_\Lambda^{(1)}, \quad (30)$$

where

$$S_\Lambda^{(1)} = -\frac{1}{2} \ln(CT_0^2), \quad (31)$$

is the leading logarithmic correction and C is the heat capacity defined as

$$C = T_0 \frac{\partial S_\Lambda^{(0)}}{\partial T_0}. \quad (32)$$

It is a matter of calculation to show that

$$C = -24\pi^2 (r_A^0)^4 M_P^2 H_0^2 \Omega_\Lambda^0 (1 + \omega_\Lambda^0). \quad (33)$$

In addition we get

$$S_\Lambda^{(1)} = -\frac{1}{2} \ln \left[-6(r_A^0)^2 M_P^2 H_0^2 \Omega_\Lambda^0 (1 + \omega_\Lambda^0) \right]. \quad (34)$$

It is easy to show that

$$dS_\Lambda = 24\pi^2 r_A^3 M_P^2 H^2 \Omega_\Lambda \left[(1 + \omega_\Lambda) dr_A - \frac{2r_A}{3na\gamma_n} \left(\frac{\Omega_\Lambda}{\gamma_n} \right)^{1/2} \left(2\gamma_n - 1 - \frac{\alpha H^2}{3M_P^2 n^4} \frac{\Omega_\Lambda}{\gamma_n} \right) dx \right], \quad (35)$$

where

$$1 + \omega_\Lambda = \frac{1}{24\pi^2 r_A^3 M_P^2 H^2 \Omega_\Lambda} \left(\frac{dS_\Lambda^{(0)}}{dr_A} + \frac{dS_\Lambda^{(1)}}{dr_A} \right) + \frac{2r_A}{3na\gamma_n} \left(\frac{\Omega_\Lambda}{\gamma_n} \right)^{1/2} \left(2\gamma_n - 1 - \frac{\alpha H^2}{3M_P^2 n^4} \frac{\Omega_\Lambda}{\gamma_n} \right) \frac{dx}{dr_A}. \quad (36)$$

Substituting the expressions for the volume, energy, and temperature in Eq. (26) for the interacting case, we get

$$dS_\Lambda = -24\pi^2 r_A^4 M_P^2 H^2 \Omega_\Lambda \left[(1 + \omega_\Lambda) \left(dx - \frac{dr_A}{r_A} \right) + \frac{Q}{9M_P^2 H^3 \Omega_\Lambda} dx \right], \quad (37)$$

where

$$1 + \omega_\Lambda = - \left[\frac{1}{24\pi^2 r_A^4 M_P^2 H^2 \Omega_\Lambda} \left(\frac{dS_\Lambda^{(0)}}{dr_A} + \frac{dS_\Lambda^{(1)}}{dr_A} \right) + \frac{Q}{9M_P^2 H^3 \Omega_\Lambda} \frac{dx}{dr_A} \right] \left(\frac{dx}{dr_A} - \frac{1}{r_A} \right)^{-1}. \quad (38)$$

Using Eq. (21) one finds

$$\frac{dS_\Lambda^{(0)}}{dr_A} = \frac{\partial S_\Lambda^{(0)}}{\partial r_A^0} \frac{dr_A^0}{dr_A} + \frac{\partial S_\Lambda^{(0)}}{\partial x^0} \frac{dx^0}{dr_A}, \quad (39)$$

where

$$\frac{dr_A^0}{dx^0} = \frac{1 + \Omega_k^0 + q_0}{H_0(1 + \Omega_k^0)^{3/2}}, \quad (40)$$

$$\frac{dr_A^0}{dr_A} = \frac{dr_A^0/dt}{dr_A/dt} = \frac{H_0 dr_A^0/dx^0}{H dr_A/dx} = \left(\frac{1 + \Omega_k^0 + q_0}{1 + \Omega_k + q} \right) \left(\frac{1 + \Omega_k}{1 + \Omega_k^0} \right)^{3/2}, \quad (41)$$

$$\frac{dx^0}{dr_A} = \frac{dx^0/dt}{dr_A/dt} = \frac{H_0 dx^0/dx^0}{H dr_A/dx} = \frac{H_0(1 + \Omega_k)^{3/2}}{1 + \Omega_k + q}, \quad (42)$$

and $q = -1 - H^{-1}dH/dx$ is the deceleration parameter. Thus we can rewrite Eq. (39) as

$$\frac{dS_\Lambda^{(0)}}{dr_A} = 24\pi^2 (r_A^0)^3 M_P^2 H_0^2 \Omega_\Lambda^0 \left[\frac{(1 + \omega_\Lambda^0)(1 + \Omega_k)^{3/2}}{1 + \Omega_k + q} \right] \left[\frac{1 + \Omega_k^0 + q_0}{(1 + \Omega_k^0)^{3/2}} - H_0 r_A^0 \right]. \quad (43)$$

In a similar manner we have

$$\frac{dS_\Lambda^{(1)}}{dr_A} = -\frac{1}{2} \frac{(1 + \Omega_k)^{3/2}}{(1 + \Omega_k + q)} \frac{d}{dt} \ln \left[(r_A^0)^2 H_0^2 \Omega_\Lambda^0 (1 + \omega_\Lambda^0) \right]. \quad (44)$$

Therefore, the interaction term may be obtained as

$$\frac{Q}{9M_P^2 H^3 \Omega_\Lambda} = (1 + \omega_\Lambda) \left(\frac{1}{r_A} \frac{dr_A}{dx} - 1 \right) - \left(\frac{\frac{dr_A}{dx}}{24\pi^2 r_A^4 M_P^2 H^2 \Omega_\Lambda} \right) \left(\frac{dS_\Lambda^{(0)}}{dr_A} + \frac{dS_\Lambda^{(1)}}{dr_A} \right), \quad (45)$$

which can also be written as

$$\begin{aligned} \frac{Q}{9M_P^2 H^3 \Omega_\Lambda} &= \frac{q(1 + \omega_\Lambda)}{1 + \Omega_k} - \frac{(r_A^0)^3 H_0^2 \Omega_\Lambda^0 (1 + \omega_\Lambda^0) q_0}{r_A^4 H^3 \Omega_\Lambda (1 + \Omega_k^0)^{3/2}} \\ &\quad + \frac{1}{48\pi^2 r_A^4 M_P^2 H^3 \Omega_\Lambda} \frac{d}{dt} \ln \left[(r_A^0)^2 H_0^2 \Omega_\Lambda^0 (1 + \omega_\Lambda^0) \right]. \end{aligned} \quad (46)$$

In this way we provide the relation between the interaction term of the dark components and the thermal fluctuation.

4 Concluding remarks

In this paper, we studied the model of interacting ECNADE by considering the entropy corrections to the NADE model. These corrections are motivated from the loop quantum cosmology which is one of the promising theories of quantum gravity. We restricted our study to the leading order correction which contains the logarithmic of the area, however, one can improve this study by considering higher order corrections to get better thermodynamic interpretation. We provided a thermodynamical description of the ECNADE model in a universe with spacial curvature. We assumed that in the absence of a coupling, the two dark components remain in separate thermal equilibrium with the horizon and that the presence of a small coupling between them can be described as stable fluctuations around equilibrium. Finally, resorting to the logarithmic correction to the equilibrium entropy we derived an expression for the interaction term in terms of a thermal fluctuation.

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